A Dynamic Search and Matching Model of Foster Care: Theory and Evidence *

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Abstract

This paper examines the two-sided dynamic matching problem in the US foster care system, where foster parents and children form reversible matches that may separate, persist, or transition to adoption. We first present new empirical facts on match transitions using administrative data. We then develop a two-sided search and matching model to rationalize these facts and generate predictions. Our findings show that a financial penalty on adoption exacerbates the intrinsic disadvantage faced by children with greater care needs and discourages high-quality matches from transitioning to adoption, highlighting important policy implications for improving permanency outcomes in foster care.

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1 Introduction

Each year, over 500,000 children spend time in the U.S. foster care system—a $30 billion federal program that provides temporary out-of-home care for children removed from their homes due to abuse, neglect, or other serious circumstances.[1] Children are placed in *institutional care* or *foster family homes*, often experiencing multiple placement changes.[2] Although foster care aims to be temporary, many children cannot reunite with their birth families and become eligible for adoption.[3] Yet, nearly 20,000 children age out of the system each year without finding a permanent family—facing steep challenges, including low college attainment and elevated risks of homelessness. Although adoption is both less costly and yields better outcomes for children,[4] foster parents often face a financial penalty when adopting, as state payments typically decrease or end, and adoptive parents assume responsibility for medical and educational expenses previously covered during foster care.

This paper studies, both theoretically and empirically, the two-sided dynamic matching process in the U.S. foster care system. In this market, foster parents and foster children form reversible matches, which may separate, persist, or transition to permanency through adoption. We first present new empirical evidence on match transitions and adoption exits among foster children. Building on these facts, we develop a two-sided search and matching model where children are heterogeneous in their level of care needed, search continues while matched to another parent, matches vary in quality, and parents face a financial penalty upon adopting. The model explains why certain children are less likely to be adopted, why others experience more frequent match separations, and how the adoption penalty differentially impacts children depending on their care needs and match quality.

In our model, adoption decisions are shaped by a key trade-off: parents must weigh

[1] A child can enter foster care for reasons such as sexual or physical abuse, parental addiction, incarceration, inability to provide care, death, inadequate housing, abandonment, behavioral problems, or addiction.

[2] Foster homes are private residences licensed to provide 24-hour care in a family-based environment. Institutional care refers to licensed facilities, such as group homes and shelters, serving multiple children at once.

[3] By federal law, if a child has been in foster care for at least 15 of the past 22 months, proceedings to terminate parental rights must begin; judges may also terminate rights earlier if it serves the child's best interests.

[4] Adoption is preferable to long-term care because it is less expensive (Barth, 1993; Barth et al., 2006; Hansen, 2008) and associated with better social and educational outcomes (Triseliotis, 2002; Hansen, 2008).

accepting the adoption penalty against securing permanence and eliminating the risk of future match separations. Our results suggest that policies aiming to limit match disruptions might be counterproductive for promoting adoption, as separations can influence parents' incentives to adopt. The model predicts that the adoption penalty exacerbates the disadvantage faced by children who require higher levels of care—who are less preferred by parents—and discourages high-quality matches from transitioning to adoption.

Using a rich panel dataset covering the universe of children relinquished for adoption in the U.S. foster care system from 2016 to 2022, we first document four stylized facts related to disability status: (**Fact 1**) it *decreases* the probability that a child transits to permanency via adoption; (**Fact 2**) it *increases* the probability that a foster placement separates; (**Fact 3**) it *decreases* the probability that a child moves from institutional care to a foster home (becomes foster matched); and (**Fact 4**) it *increases* the probability that a child moves from a foster home to institutional care (becomes unmatched). Although the empirical specification and theoretical model could also be used to study other child characteristics, such as gender, race, or ethnicity,[5] we focus on disability for two reasons. First, most policy efforts aimed at increasing adoption specifically target children with disabilities. Second, the adoption penalty may be more severe for these children, as adoptive parents assume greater medical and care-related expenses. It is important to note that the available data do not allow us to observe match quality directly. Therefore, the theoretical model serves not only to rationalize the documented empirical facts but also to explore how unobserved match quality affects match formation, separation, and adoption transitions.

To analyze how different forces interact in agents' decisions to form a foster match, separate a foster match, and transition to permanency via adoption, we develop a dynamic matching model with search frictions and non-transferable utility. Search frictions imply that it takes time to find a match, and transfers are exogenously determined. Children and parents can form two types of matches: foster matches (reversible) and adoption matches (irreversible). We assume that (a) children are heterogeneous in their level of care needed; (b) adoption can only occur after a foster match forms; (c) parents receive a smaller per-period payoff when adoption matched than when foster

[5] The empirical specification and theoretical model can be extended to study the effect of other observable child characteristics, such as gender, race, and ethnicity.

matched; and (d) matches vary in quality. Both sides prefer matches of higher quality, and parents prefer children who require a lower level of care.

Each period, unmatched children (or foster-matched children seeking improvement) meet unmatched parents. Upon meeting, agents draw a match quality but initially observe only a noisy signal. A foster match forms if and only if both agents accept based on the signal. If a new foster match forms, any previous foster match dissolves. Once a foster match is formed, the true match quality is revealed and remains constant throughout the relationship. Given the realized quality, agents then decide whether to maintain the foster match, transition to adoption, or separate.

The theoretical model serves two purposes. First, it identifies the mechanisms underlying the empirical facts and explains why these empirical patterns arise in equilibrium. Second, it generates additional equilibrium predictions regarding the role of match quality. Regarding foster match separations, we show that the higher separation probability of children with disabilities (Fact 2) reflects two opposing mechanisms. On the one hand, foster matches involving children with disabilities are more likely to dissolve once uncertainty about match quality is resolved, increasing the likelihood of separation. On the other hand, conditional on the foster match surviving, these children are less likely to transition to another foster family in search of a better match, reducing the likelihood of separation. Fact 2 therefore implies that the first mechanism dominates in equilibrium. Because the data do not identify the reason for each foster match separation, distinguishing between these two mechanisms relies entirely on the theoretical model. In addition, the model predicts that high-quality foster matches are less likely to separate than low-quality foster matches. In contrast to the case of disability, the two sources of separation reinforce one another. Higher-quality foster matches are less likely to dissolve once uncertainty about match quality is resolved and, conditional on surviving, are also less likely to transition to another foster family in search of a better match.

Regarding adoptions, we show that the lower adoption probability of children with disabilities (Fact 1) reflects two reinforcing mechanisms. First, children with disabilities are less likely to form a foster match because foster parents require stronger signals of match quality before accepting a foster match. Second, conditional on forming a foster match, foster parents caring for children with disabilities have stronger incentives to remain in foster care rather than transition to adoption. Even absent an adoption penalty, children with disabilities are less likely to transition to adoption for two reasons. First, they generate lower surplus from adoption. Second, conditional on the foster match surviving, they are less likely to transition to another foster family in search of a better match, making continued fostering relatively more attractive. The adoption penalty amplifies this disadvantage, further reducing their likelihood of adoption. The same mechanism also generates a novel prediction regarding match quality. As match quality improves, foster matches become less likely to transition to another foster family. Consequently, in the presence of an adoption penalty, adoption decisions need not be monotonic in match quality: some relatively high-quality foster matches may optimally remain in foster care even though lower-quality matches transition to adoption.

Related Literature. This paper contributes to the growing literature analyzing foster care as a matching market. Slaugh et al. (2016) studies the Pennsylvania Adoption Exchange, showing that better information and centralized matching tools improve adoption rates. Robinson-Cortés (2021) finds that thicker markets and cross-regional placements stabilize foster care matches. Dierks et al. (2021) develops a dynamic search-and-matching model, highlighting the advantages of caseworker-driven search strategies. On the design side, Altınok and MacDonald (2025) propose licensing menus to screen foster parents under adverse selection and search frictions, while Dierks et al. (2024) designs incentive-compatible platforms for children with disabilities. Baron et al. (2024) extends matching ideas to case assignment, improving outcomes without harming investigators. Finally, Highsmith (2024) develops dynamic mechanisms that promote timely and efficient placements, outperforming traditional sequential matching. Our paper stands out by combining new empirical evidence on match transitions with a theoretical two-sided search and matching model. Unlike prior work that focuses on one-time placement decisions or matching tool design, we emphasize the dynamics of reversible matches and show how financial penalties on adoption distort match quality, particularly disadvantaging children with greater care needs. In doing so, we shift the focus from initial match efficiency to long-term permanency outcomes in foster care.

We also contribute to the literature analyzing the role of financial incentives in adoption decisions. Empirical studies document that reducing adoption penalties raises adoption rates. For example, Bishop and MacDonald (2024) and Simon et al. (2024) analyze a policy change in Minnesota that eliminated the financial penalty for adopting children aged six and older, finding that adoption probabilities increased after the reform. Similarly, Argys and Duncan (2012) show that a smaller difference between foster care and adoption payments leads to higher adoption rates. Our contribution differs by focusing on the types of matches that transition to adoption, providing theoretical predictions about how financial penalties distort match quality and affect which children achieve permanency.

Beyond its application to foster care, our paper contributes to the broader literature on dynamic matching with heterogeneous agents. Most existing models assume that once matches are formed, they do not endogenously dissolve, focusing instead on issues such as dynamic stability in irreversible settings (Altınok, 2025; Doval, 2022), the welfare implications of matching algorithms (Leshno, 2022; Baccara et al., 2020; Akbarpour et al., 2020; Anderson et al., 2015; Ünver, 2010), and the emergence of positive assortative matching (Smith, 2006; Chade, 2006; Shimer and Smith, 2000; Eeckhout, 1999; Burdett and Coles, 1997). In contrast, the small literature that allows for reversibility typically adopts a cooperative framework in which agents anticipate that current or future partners may exit and coordinate on credible re-matching plans (Liu, 2023; Kurino, 2020; Kadam and Kotowski, 2018; Damiano and Lam, 2005). Our paper departs from these approaches by modeling both reversible and irreversible match transitions within a unified framework. We document new empirical patterns of match persistence, dissolution, and adoption, and establish sufficient conditions under which these dynamics emerge endogenously in equilibrium.

Organization of the Paper. The rest of the paper is organized as follows. Section 2 presents the empirical facts that motivate the theoretical model. Section 3 presents the model, describing the environment, timing, and strategies. Section 4 develops the recursive formulation and defines the equilibrium. Section 5 analyzes the equilibrium outcomes, characterizes the conditions under which the equilibrium is consistent with the empirical facts, and derives predictions regarding match quality. Lastly, Section 6 concludes. All proofs are contained in the Appendix.

2 Empirical Analysis

We motivate the key features of our two-sided dynamic matching model through an empirical analysis.[6] Using administrative data covering the universe of children in the U.S. foster care system from 2016 to 2022, we document new stylized facts about the matching process between foster children and foster parents.

2.1 Data and Descriptive Statistics

We use the 6-month Foster Care Files from the Adoption and Foster Care Analysis and Reporting System (AFCARS),[7] a comprehensive administrative database covering all children in the U.S. foster care system from 2016 to 2022. The data are structured as an unbalanced panel, with observations recorded every six months for each child from entry into foster care until exit. Exits occur through reunification with the birth family, adoption, emancipation, guardianship, transfer to another agency, running away, or death, with both the manner and date of exit recorded. The dataset provides a rich set of child-level variables,[8] including gender, race and ethnicity, disability status, Title IV-E funding eligibility,[9] date of birth, date of most recent entry into foster care, and, if applicable, the date of termination of parental rights.[10]

In the data, the disability variable—central to our empirical analysis—indicates whether a child has been clinically diagnosed with a disability, clinically diagnosed without a disability, or has not yet been diagnosed. Disabilities encompass a wide range of conditions, including blindness, glaucoma, arthritis, multiple sclerosis, Down syndrome, personality disorders, attention deficit disorder, and anxiety disorder, among others. We classify a **child as having a disability** if she has been clinically diagnosed with at least one condition; otherwise, the child is considered not to have a disability. Although the majority of children receive a mandatory medical evaluation upon entry

[6] The goal of the empirical analysis is not to establish causality, but rather to document robust correlations while controlling for a rich set of covariates.

[7] AFCARS is a federally mandated data collection system. All fifty U.S. states and the District of Columbia are required to collect data on all children in foster care and all children adopted from foster care.

[8] Following Buckles (2013) and Brehm (2017), we use the most recent record for each child, which updates demographic information.

[9] Title IV-E is a federal program through which states receive reimbursement for payments made on behalf of eligible children.

[10] To protect confidentiality, dates of birth are standardized to the 15th of the month, and all dates are recoded to preserve consistent time intervals.

into the foster care system, the data do not allow us to identify the specific type, number, or severity of diagnosed conditions. Accordingly, we assume that disability status reflects pre-existing conditions.[11]

For each period (semester) that children remain in foster care, the data report their most recent placement and the placement's start date. Placement types are classified as pre-adoptive home, non-relative foster home, relative foster home, group home, institution, supervised independent living, and runaway. We define children as **foster matched** in a given period if they are placed in a pre-adoptive home, non-relative foster home, or relative foster home.[12] Conversely, we define children as **unmatched** if they are placed in a group home or an institution.

To maintain a consistent estimation sample, we restrict the data to children younger than 16 whose parental rights have been terminated. The age restriction excludes older children who are more likely to exit foster care through legal emancipation, while the parental rights restriction ensures that all children in the sample are eligible for adoption. We further restrict the sample to children who are either foster matched or unmatched, as defined earlier. This yields a full child-level sample of 396,823 children (Sample A). From this full sample, we construct two child-period subsamples. The first (Sample B) includes periods in which a child is foster matched at the beginning of the period and remains in foster care at the end; the second (Sample C) includes periods where a child is unmatched at the beginning and remains in foster care at the end. Table A1 summarizes key statistics for the full sample and both subsamples. Table A2 reports the same statistics conditional on the child's disability status, the main variable of interest.

In Table A1 (Sample A), the average age of children is almost 8 years, and they have had their parental rights terminated for approximately 20 months. Among all children, 36 percent have been diagnosed with a disability. In a given period, 95 percent of children are foster matched, with the average duration of a foster match being 21 months. We define a child as **adopted** if she exits the system through adoption, with 26 percent

[11] This is a strong assumption, as the presence or severity of a disability could potentially vary with the amount of time spent in group care or the quality of foster placements. Ideally, we would treat disability as a time-varying characteristic, but the data do not permit us to observe changes in disability status over time.

[12] Although foster parents are not individually identifiable, information on household structure, foster parents' race, and year of birth is reported.

of children adopted in a given period. Conditional on being foster matched at the beginning of a period, we define a child as **becoming unmatched** if she is unmatched at the end of that same period; in Sample B, the probability of becoming unmatched is 2 percent. Conversely, conditional on being unmatched at the beginning of a period, a child is defined as **becoming foster matched** if she is foster matched at the end; in Sample C, the probability of this transition is 27 percent.

Transitions in and out of matches are influenced not only by entry into or exit from foster care but also by separations from existing foster matches. We define a **foster match separation** as occurring when a child who is foster matched at the beginning of a period is no longer placed with the same foster parent at the end of the period.[13] Table A1 (Sample B) shows that such separations occur with a probability of 18 percent. Separations can arise for various unobserved reasons, such as a social worker moving the child to institutional care, a foster parent requesting removal, or a transfer to a more suitable placement. Unfortunately, the dataset does not record the specific cause of each separation.

2.2 Empirical Specifications and Stylized Facts

We estimate the association between disability status and four key outcomes: **(1)** the probability that a child is adopted, **(2)** the probability that a foster match separates, **(3)** the probability that a child transitions into a foster match (i.e., becomes foster matched), and **(4)** the probability that a child transitions out of a foster match (i.e., becomes unmatched). For each outcome, we estimate the following linear probability model:

$$y_{ijt} = \alpha + \gamma \, disability_i + \beta X_i + \theta Z_{it} + \lambda_t + \xi_j + \epsilon_{ijt} \tag{1}$$

where y_{ijt} is an indicator for the outcome of child i in state j at period t. The variable $disability_i$ is an indicator variable indicating whether child i has been diagnosed with a disability. X_i is a vector of time-invariant child characteristics, including gender, race, ethnicity, and Title IV-E eligibility. Z_{it} captures time-varying characteristics, such as age in months, months spent in foster care, and months since parental rights were terminated. We include period fixed effects λ_t to control for time trends and state fixed

[13] Although foster parents are not individually identifiable, a variable tracking the number of placements allows us to infer whether the child has been placed with a different caregiver.

Table 1: Stylized Facts from Foster Care - Effect of Disability

	Adoption I	Foster Matched Placement II	Foster Match Separation III	Becomes Foster Matched IV	Becomes Unmatched V
Disability γ	-0.015*** (0.003)	-0.042*** (0.002)	0.027*** (0.003)	-0.048*** (0.006)	0.011*** (0.001)
Mean of dependent variable	0.264	0.946	0.176	0.273	0.016
Sample	A	A	B	C	B

Notes: Data are from the Adoption and Foster Care Analysis and Reporting System (AFCARS). All specifications control for child demographics, state indicators, and period indicators. Columns I and II use Sample A, Columns III and IV uses Sample B, and Column IV uses Sample C. Standard errors are clustered at the state-period level and shown in parentheses. ***$P < 0.01$; **$P < 0.05$; *$P < 0.10$.

effects ξ_j to account for unobserved, time-invariant differences across states.

Fact 1: Disability Decreases the Probability of Being Adopted. Adoption rates among children with and without disabilities are 23 and 28 percent, respectively (see Table A2). To assess whether this difference persists after controlling for observable characteristics, we use Sample A to estimate Equation 1, where the dependent variable y_{ijt} equals one if child i in state j is adopted in period t, and zero otherwise. Column I of Table 1 shows that children with disabilities have a 2 percentage point lower probability of being adopted than children without disabilities—a difference that is statistically significant and robust to the inclusion of controls.

One potential mechanism is that many states require prospective adoptive parents to first foster a child. If children with disabilities are less likely to be foster matched initially, this may partly explain their lower adoption rates. To investigate this possibility, we re-estimate Equation 1 redefining the dependent variable y_{ijt} to equal one if child i in state j is foster matched in period t. As shown in Column II of Table 1, the coefficient on disability is negative, indicating that children with disabilities are indeed less likely to be foster matched. Although this result is suggestive, the theoretical model provides further insight by decomposing the adoption gap: children with disabilities are not only less likely to be foster matched, but conditional on being foster matched, they are also less likely to transition to adoption.

Fact 2: Disability Increases the Probability of Foster Match Separation. In the data, foster matches involving children with and without disabilities separate at rates of 18 and 17 percent, respectively (see Table A2). To assess whether this difference persists after controlling for observable characteristics, we use Sample B to estimate Equation 1, where the dependent variable y_{ijt} equals one if child i in state j experiences a foster match separation in period t, and zero otherwise. In this specification, the vector Z_{it} includes the number of months the child has spent in the current foster match and an indicator for placement type (i.e., whether the child is placed in a pre-adoptive home, a non-relative foster home, or a relative foster home).

Table 1, Column III, shows that children with disabilities are 3 percentage points more likely to experience a foster match separation than children without disabilities. Although the dataset does not allow us to observe the specific cause of each separation, our theoretical model distinguishes between two types: (a) transitions from a foster match to being unmatched (i.e., from a foster home to institutional care) and (b) transitions from one foster match to another (i.e., moving between foster homes). To interpret the empirical pattern, the model shows that these two forces operate in opposite directions: children with disabilities are more likely to experience the first type of separation and less likely to experience the second.

Fact 3: Disability Decreases the Probability of Becoming Foster Matched. The rates of foster match formation—conditional on being unmatched at the beginning of the period—are 26 and 31 percent for children with and without disabilities, respectively (see Table A2). To examine the effect of disability on the probability of transitioning into a foster match, we use Sample C to estimate Equation 1, where the dependent variable y_{ijt} equals one if child i in state j becomes foster matched in period t, and zero otherwise. In this specification, the vector Z_{it} additionally includes the number of months the child has spent unmatched and the type of current placement (group home or institution).

Table 1, Column IV, shows that children with disabilities are 5 percentage points less likely to transition into a foster match than children without disabilities. To interpret this result, our theoretical model highlights two mechanisms: disability reduces the probability that a child is successfully matched with a foster parent, and more broadly, conditional on forming a match, it increases the risk that the match dissolves.

Fact 4: Disability Increases the Probability of Becoming Unmatched. In the data, 3 percent of children with disabilities and 1 percent of children without disabilities transition into unmatched status, conditional on starting the period foster matched (see Table A2). To examine whether this difference persists after controlling for observables, we use Sample B to estimate Equation 1, where the dependent variable y_{ijt} equals one if child i in state j becomes unmatched in period t, and zero otherwise. As in the previous specification, the vector Z_{it} includes the number of months the child has spent in the current foster match and an indicator for the type of foster placement.

As shown in Table 1, Column V, children with disabilities are significantly more likely to transition into unmatched status. In our theoretical framework, this probability reflects both the likelihood that a foster match separates and the probability that the child finds a new foster parent. As with separations, these two forces can operate in opposite directions: children with disabilities are more likely to experience a separation and less likely to be re-matched, which results in a higher probability of becoming unmatched.

3 Model

In this section, we develop a search and matching model to analyze how various incentives influence agents' decisions regarding match formation and separation. The theoretical model serves two purposes. First, it enhances our understanding of the empirical patterns documented in Section 2. Second, it allows us to examine how transitions in children's matches are influenced by match quality—an aspect that is unobservable to the econometrician.

3.1 Environment

Time is discrete and the horizon is infinite. One side of the market consists of **children**, who differ in an observable attribute $x \in [\underline{x}, \overline{x}] \equiv X \subset \mathbb{R}$, where x denotes the inverse level of care the child requires, so that higher values correspond to lower care needs. Each period, a strictly positive mass ρ of children enters the market, and each child draws an attribute from the full-support distribution $l(x)$. The other side of the market consists of homogeneous **parents**. The mass of parents outside the market is strictly positive, and parents decide each period whether to enter or exit the market.

Children and parents who are in the market can be unmatched or matched. Let

$u_t^p \geq 0$ and $u_t^c(x)$ denote the endogenous period-t masses of unmatched parents and children in the market, respectively. Matches are one-to-one, formed between children and parents, and heterogeneous in **quality**, denoted by $q \in [\underline{q}, \overline{q}] \equiv Q \subset \mathbb{R}$.[14]

There are two types of matches: **foster matches** (reversible) and **adoption matches** (irreversible). Agents who form a foster match (hereafter *f-match*) remain in the market, while agents who form an adoption match (hereafter *a-match*) leave the market. Let $m_t(x, q)$ denote the endogenous distribution of foster matches. Thus, the aggregate state of the market at period-t is summarized by $\phi_t = (u_t^p, u_t^c, m_t)$.[15]

All agents are risk-neutral and discount the future at rate $\beta \in (0, 1)$. An unmatched child's payoff is normalized to zero. A child and a parent in match (x, q, z), respectively, receive flow payoffs $b^c(x, q, z)$ and $b^p(x, q, z)$, where $z \in \{f, a\}$ indicates whether the match is foster or adoption. Parents' payoffs outside the market are normalized to zero, and they incur a per-period cost $k > 0$ to hold a license and remain in the market. .

The following assumption is maintained throughout the paper.

Assumption 1 (**Payoffs**)**.** *For each $z \in \{f, a\}$: **(i)** $b^c(x, q, z)$ and $b^p(x, q, z)$ are continuous; **(ii)** $b^c(x, q, z)$ and $b^p(x, q, z)$ are increasing in q; **(iii)** $b^p(x, q, z)$ is increasing in x; **(iv)** $b^c(x, q, a) > b^c(x, q, f)$ for every (x, q); **(v)** $b^p(x, q, f) \geq b^p(x, q, a)$ for every (x, q).*

Assumption 1 states that (ii) both agents prefer higher-quality matches, (iii) parents prefer children with lower care needs, and (iv) children strictly prefer adoption to fostering at a given match quality. These properties ensure that higher-quality matches generate greater surplus for both agents and that parents are prefer to match with children requiring lower levels of care. The condition (v) $b^p(x, q, f) \geq b^p(x, q, a)$ allows adoption to yield lower per-period payoff for parents than fostering. When the inequality holds with equality, adoption and fostering yield identical payoffs to parents, corresponding to the benchmark case without an adoption penalty. When the inequality is strict, the difference captures the *adoption penalty*, reflecting institutional and financial factors that make adoption less attractive than fostering, such as lower subsidies and greater long-term financial responsibility. Additional assumptions used only for particular comparative statics results are introduced in the Appendix.

[14] Match quality may capture factors affecting the relationship beyond the child's observable type, such as the emotional bond between the child and the parent or the relationship between the parent and the child's birth family.

[15] To simplify notation, we suppress the time subscript t whenever no confusion arises, as our analysis focuses on the stationary equilibrium.

3.2 Timing and Strategies

Each period is divided into four stages, as illustrated: (1):

Figure 1: Timeline

(i) **Destruction and a-matching stage.** Any child is adopted by a relative with exogenous probability $\delta \in (0,1)$.[16] Children adopted by a relative receive $b^c(x, \overline{q}, a)$, which captures that relative placements represent the highest possible match quality. The f-match breaks if a child is adopted exogenously. Now, if the f-match remains, then child and parent announce simultaneously *adoption*, *destroy*, or *remain*. An f-match destroys if at least one agent announces destroy, and an a-match takes place if and only if both agents announce adoption. If an f-match is destroyed, the parent remains unmatched that period and the child searches. Agents who form an a-match receive adoption payoffs to perpetuity, and we assume that q remains constant when transitioning from f-matched to a-matched.

(ii) **Entry and Exit stage.** A mass of new children enters the market and parents make entry/exit decisions. Parents and children who enter the market remain unmatched that period. Agents who formed an a-match during the previous stage leave the market, and only unmatched parents can decide to exit the market.

(iii) **Search and f-matching stage.** Children search when unmatched or f-matched, and parents search only when unmatched. Meetings are stochastic and are described by the market tightness $\theta \in \mathbb{R}_+$ (i.e. parents-to-children ratio):

$$\theta = \frac{u^p}{\int_x \left\{ u^c(x) + \int_q m(x,q)\mathrm{d}q \right\} \mathrm{d}x}.$$

A child meets a parent with probability $\pi^c(\theta)$ which is a strictly increasing and strictly concave function such that $\pi^c(0) = 0$. Similarly, a parent meets a child

[16] In some cases, relatives reach out when they learn about the situation and request to adopt the child. Child welfare agencies have strong preferences for relatives.

with probability $\pi^p(\theta)$ which is a strictly decreasing and convex function such that $\pi^p(\theta) = \frac{\pi^c(\theta)}{\theta}$ and $\pi^p(0) = 1$.

Next, when a child and parent meet, a match quality q is drawn from the full-support probability distribution $r(q)$. Before forming an f-match, agents observe a noisy signal $s \in [\underline{s}, \overline{s}] \equiv S \subset \mathbb{R}$ about match quality. Conditional on the signal, match quality follows the full-support density $g(q \mid s)$. Let $G(q \mid s)$ denote the corresponding cumulative distribution function, and let $h(s)$ denote the distribution of signals.

Assumption 2 (**Informative Signals**). *The conditional distribution of match quality given signal s satisfies first-order stochastic dominance: for any $s', s \in S$ with $s' > s$,*

$$G(q \mid s') \leq G(q \mid s) \quad \textit{for all } q \in Q.$$

After observing the noisy signal, agents announce simultaneously *foster* or *reject*. An f-match is formed if and only if both agents announce foster. If a (new) f-match is formed, then any old f-match dissolves. Upon formation of an f-match, the true match quality q becomes known and remains constant for the duration of the match.

(iv) **Payoff stage.** Upon formation of an f-match, agents perfectly observe the quality q. Given complete information about match quality, the payoffs over the remaining lifetime of the f-match are known.

We restrict attention to *stationary, pure, and symmetric Markov* strategies that depend only on the stationary aggregate market state, $\phi = (u^p, u^c, m)$.[17] For simplicity, we suppress the explicit dependence on ϕ in the notation.

Each parent's strategy is a tuple (in, out, f^p, d^p, a^p). The variables $in \in \{\text{no}, \text{yes}\}$ and $out \in \{\text{no}, \text{yes}\}$ denote the parent's entry and exit decisions, respectively. The function $f^p : X \times S \to \{0, 1\}$ specifies whether the parent forms an f-match with child x after observing signal s. The function $d^p : X \times Q \to \{0, 1\}$ determines whether an existing f-match with child x at quality level q is destroyed, while $a^p : X \times Q \to \{0, 1\}$ governs

[17] For a more detailed description of the stationary aggregate market state, see Appendix B.2.

the adoption decision. Throughout the paper, binary choice variables take value 1 if the corresponding action is taken (foster, destroy, or adopt) and 0 otherwise.

For each child x, a strategy consists of the triple (f^c, d^c, a^c). The functions $d^c : X \times Q \to \{0,1\}$ and $a^c : X \times Q \to \{0,1\}$ represent the decisions to destroy and to adopt for child (x, q), respectively. Let $\tilde{Q} = Q \cup \{q_0\}$ denote the extended quality set, where q_0 represents the unmatched state and $\tilde{q} \in \tilde{Q}$ denotes a generic element of this set. Then, $f^c : X \times \tilde{Q} \times S \to \{0,1\}$ determines whether to form a new f-match after child $(x, \tilde{q})$ observes signal s.

Next, we define the decision to create a new foster match, dissolve an existing one, and transition into adoption as correspondences.

Definition 1. *A **foster-matching, dissolution,** and **adoption** are correspondences $\mathcal{F} : X \times \tilde{Q} \rightrightarrows S$, $\mathcal{D} : X \rightrightarrows Q$, and $\mathcal{A} : X \rightrightarrows Q$ such that:*

(i) *$s \in \mathcal{F}(x, \tilde{q})$ if and only if $f^c(x, \tilde{q}, s) = 1 = f^p(x, s)$,*

(ii) *$q \in \mathcal{D}(x)$ if and only if $d^c(x, q) = 1$ or $d^p(x, q) = 1$, and*

(iii) *$q \in \mathcal{A}(x)$ if and only if $a^c(x, q) = 1 = a^p(x, q)$, respectively.*

4 Recursive Formulation and Equilibrium

In this section, we characterize the stationary equilibrium of the model and show how its properties align with the stylized facts documented in the empirical analysis.

4.1 Children's Value Functions

We define $\hat{\mathcal{C}}(x, \tilde{q})$ as the search value function of a child $(x, \tilde{q})$ at the beginning of the search and f-matching stage. Similarly, $\mathcal{C}(x, q)$ denotes the value function of a child (x, q) at the end of the payoff realization stage, once an f-match has occurred. The search value is thus given by:

$$\hat{\mathcal{C}}(x, \tilde{q}) = \Big(1 - \pi^c(\theta) \int\limits_{\mathcal{F}(x,\tilde{q})} h(s)\mathrm{d}s\Big)\mathcal{C}(x, \tilde{q}) + \pi^c(\theta) \int\limits_{\mathcal{F}(x,\tilde{q})} \mathbb{E}_s[\mathcal{C}(x, q')]h(s)\mathrm{d}s \qquad (2)$$

where $\mathbb{E}_s[\mathcal{C}(x,q')] \equiv \int_Q \mathcal{C}(x,q')\mathrm{d}G(q'|s)$ is the conditional expected value given signal s. In short, a child meets a parent with probability $\pi^c(\theta)$, and then a new f-match forms conditional on the signal. Therefore, the child exits the search and f-matching stage either in the same state as she entered, receiving $\mathcal{C}(x,\tilde{q})$, or with a new f-match of an unknown quality q' and receives an expected value conditional on the signal s.

Now, the value function for an unmatched child x at the end of the payoff realization stage is:

$$\mathcal{C}(x,q_0) = \beta\delta\frac{b^c(x,\overline{q},a)}{1-\beta} + \beta(1-\delta)\hat{\mathcal{C}}(x,q_0) \tag{3}$$

Analogously, the value function for a child who is f-matched at the end of the payoff realization stage is:

$$\begin{aligned}\mathcal{C}(x,q) = b^c(x,q,f) + \beta\delta\frac{b^c(x,\bar{q},a)}{1-\beta} + \beta(1-\delta)\Big[&d^p(x,q)\hat{\mathcal{C}}(x,q_0)\\ &+ \big(1-d^p(x,q)\big)a^p(x,q)\ \max\left\{\frac{b^c(x,q,a)}{1-\beta}, \hat{\mathcal{C}}(x,q_0), \hat{\mathcal{C}}(x,q)\right\}\\ &+ \big(1-d^p(x,q)\big)\big(1-a^p(x,q)\big)\ \max\left\{\hat{\mathcal{C}}(x,q_0), \hat{\mathcal{C}}(x,q)\right\}\Big]\end{aligned} \tag{4}$$

In words, in the current period, she receives the f-match payoff $b^c(x,q,f)$. At the beginning of the next period, she is adopted by a relative with probability δ. If the f-match remains, child and parent decide between transit to an a-match, destroy the f-match, or remain f-matched. In each case, the possible continuation values for child (x,q) are $\frac{b^c(x,q,a)}{1-\beta}$, $\hat{\mathcal{C}}(x,q_0)$, and $\hat{\mathcal{C}}(x,q)$, respectively.

Next, we characterize children's foster matching, adoption, and match destruction decisions in terms of their value functions.

Lemma 1. ***(i)*** *For each child* $(x,\tilde{q})$, $f^c(x,\tilde{q},s) = 1$ *if and only if* $\mathbb{E}_s[\mathcal{C}(x,q')] \geq \mathcal{C}(x,\tilde{q})$. *Moreover, for each child* (x,q), ***(ii)*** $a^c(x,q) = 1$ *if and only if* $\frac{b^c(x,q,a)}{1-\beta} \geq \max\{\hat{\mathcal{C}}(x,q_0), \hat{\mathcal{C}}(x,q)\}$, *and* ***(iii)*** $d^c(x,q) = 1$ *if and only if* $\hat{\mathcal{C}}(x,q_0) > \max\{\frac{b^c(x,q,a)}{1-\beta}, \hat{\mathcal{C}}(x,q)\}$.

These decision rules arise from simple comparisons of continuation values. Whenever a child receives a foster opportunity, she accepts it if and only if the expected continuation value implied by the observed signal weakly exceeds the value of her current state, whether unmatched or already f-matched.

Once in an f-match, the child compares the value of adoption, continued fostering, and re-entering the search process. Adoption is optimal when the lifetime value of an a-match dominates the alternatives, while match destruction occurs when returning to the market yields a higher continuation value than either adoption or remaining f-matched. Otherwise, the child prefers to remain in the existing f-match.

4.2 Parents' Value Functions

As with children, we define $\hat{\mathcal{P}}(x,q)$ as the value function of a parent at the beginning of the search and f-matching stage who is f-matched with a child (x,q), and $\mathcal{P}(x,q)$ as her value function at the end of the payoff realization stage. Let $\mathcal{P}^u$ denote the value function of an unmatched parent at the beginning of the entry and exit stage. The search value is therefore given by:

$$\hat{\mathcal{P}}(x,q) = \Big(1 - \pi^c(\theta) \int\limits_{\mathcal{F}(x,q)} h(s)\mathrm{d}s\Big)\mathcal{P}(x,q) + \pi^c(\theta) \int\limits_{\mathcal{F}(x,q)} \mathcal{P}^u\, h(s)\mathrm{d}s \tag{5}$$

That is, a parent who is in an f-match with a child (x,q) leaves the search and f-matching stage either by becoming unmatched or by remaining in the current match. Now, the value function for an unmatched parent is given by:

$$\begin{aligned}\mathcal{P}^u = \max\Big\{0, -k + \beta\pi^p(\theta) \int\limits_{\mathcal{F}(x,\tilde{q})} \int\limits_{(x,\tilde{q})} \mathbb{E}_s[\mathcal{P}(x,q)]\, \hat{m}(x,\tilde{q})\, \mathrm{d}(x,\tilde{q})\, h(s)\mathrm{d}s \\ + \beta\Big(1 - \pi^p(\theta) \int\limits_{\mathcal{F}(x,\tilde{q})} \int\limits_{(x,\tilde{q})} \mathcal{P}^u\, \hat{m}(x,\tilde{q})\, \mathrm{d}(x,\tilde{q})\, h(s)\mathrm{d}s\Big)\Big\}\end{aligned} \tag{6}$$

where $\mathbb{E}_s[\mathcal{P}(x,q)] \equiv \int_Q \mathcal{P}(x,q)\mathrm{d}G(q|s)$ and $\hat{m}(x,\tilde{q})$ is the distribution of child types in the market.[18] To sum up, an unmatched parent decides between staying or exiting the market. If she exits, she gets 0 payoff, if not, she pays a cost of maintaining the license k and joins the search and f-matching stage next period. She then either forms an f-match (x,q) or continues unmatched.

[18] For a more detail specification of the distribution of children, see Appendix B.1

Lastly, the value function for an f-matched parent is given by:

$$\begin{aligned}\mathcal{P}(x,q) = b^p(x,q,f) + \beta\delta\mathcal{P}^u + \beta(1-\delta)\Big[& d^c(x,q)\mathcal{P}^u \\ &+ \big(1-d^c(x,q)\big)a^c(x,q)\max\left\{\frac{b^p(x,q,a)}{1-\beta}, \mathcal{P}^u, \hat{\mathcal{P}}(x,q)\right\} \\ &+ \big(1-d^c(x,q)\big)\big(1-a^c(x,q)\big)\max\left\{\mathcal{P}^u, \hat{\mathcal{P}}(x,q)\right\}\Big]\end{aligned} \tag{7}$$

Next, we characterize parents' foster matching, adoption, and match destruction decisions in terms of their value functions.

Lemma 2. *For a parent outside the market, **(i)** $in = yes$ if and only if $\mathcal{P}^u \geq 0$. For an unmatched parent, **(ii)** $f^p(x,s) = 1$ if and only if $\mathbb{E}_s[\mathcal{P}(x,q)] \geq \mathcal{P}^u$. Moreover, for each parent who is f-matched with a child (x,q), **(iii)** $a^p(x,q) = 1$ if and only if $\frac{b^p(x,q,a)}{1-\beta} \geq \max\{\mathcal{P}^u, \hat{\mathcal{P}}(x,q)\}$, and **(iv)** $d^p(x,q) = 1$ if and only if $\mathcal{P}^u > \max\{\frac{b^p(x,q,a)}{1-\beta}, \hat{\mathcal{P}}(x,q)\}$.*

Parents' decisions follow the same comparison logic as in the children's case. An out-of-market parent enters whenever the value of participation weakly exceeds the outside option. Upon receiving a foster opportunity, an unmatched parent accepts it if the expected continuation value implied by the signal dominates remaining unmatched. Once in an f-match, the parent compares the value of adoption, continued fostering, and the unmatched value, choosing adoption when the lifetime value of an a-match is highest, destroying the match when re-entry dominates, and otherwise remaining f-matched.

4.3 Equilibrium

We now define the equilibrium concept used in the analysis. A formal definition is provided in Appendix C.

Definition 2 (Foster-Care Equilibrium (Informal)). *A **foster-care equilibrium** consists of value functions for children and parents, individual strategies governing entry, foster matching, adoption, and match destruction, and an aggregate market state such that: **(i)** given strategies and the aggregate state, value functions satisfy the corresponding Bellman equations; **(ii)** given value functions and the aggregate state, individual strategies are optimal; and **(iii)** given the value functions and strategies, the aggregate state is stationary.*

We restrict attention to stationary equilibria in which there is a positive mass of unmatched parents in the market, i.e., $u^p > 0$. With the outside option normalized to zero, this implies a free-entry condition for parents: in equilibrium, the value of remaining in the market must be zero, $\mathcal{P}^u = 0$. If $\mathcal{P}^u$ were strictly positive, additional parents would enter the market, increasing the market tightness and reducing the expected return from search until the free-entry condition is restored.

The free-entry condition pins down the equilibrium market tightness θ^* jointly with the stationary distribution of children and matches. An explicit characterization of this condition, together with the flow-balance equations determining the stationary aggregate state, is provided in Appendix B.2.

5 Theoretical Equilibrium Analysis

Although we characterize the equilibrium through a sequence of dynamic optimization problems, the underlying economic structure is simple. A key feature of our results is that they do not depend on the specific equilibrium levels of the aggregate state $\phi = (u_p, u_c, m)$ defined in Section 3. Rather, they follow from monotonicity properties of the value functions that hold for any stationary aggregate state ϕ satisfying Definition 2. In particular, three objects summarize the equilibrium: (i) a signal threshold governing foster-match formation (Proposition 1), (ii) a match-quality threshold governing dissolution (Proposition 2), and (iii) an adoption region—possible a union of multiple match-quality intervals—governing adoption (Proposition 3).

5.1 Overview of the Characterization

Characterizing these objects requires establishing several properties of the value functions and decision rules. We collect the formal proofs in Appendix D and summarize the logic here in four steps, before turning to the economic implications in Sections 5.2 and 5.3.

The characterization proceeds in four steps:

Step 1: We establish monotonicity properties of the value functions with respect to match quality, child type, and signal realizations.

Step 2: Using these monotonicity results, we show that individual decisions admit threshold representations: fostering decisions are characterized by signal cutoffs and dissolution decisions by match-quality cutoffs. Adoption decisions are governed by match-quality cutoffs as well, but the resulting adoption region may be non-convex when adoption penalties are present.

Step 3: Combining the child's and parent's threshold rules yields the joint equilibrium decision regions.

Step 4: We derive comparative statics showing how these equilibrium regions vary with children's care needs and current match quality, and we show how an adoption penalty can render the adoption region non-convex.

5.2 Main Equilibrium Implications

We now provide the equilibrium decision rules for match formation, dissolution, and adoption. We begin with the joint fostering decisions:

Proposition 1 (**Foster-Match Decisions**)**.** *In equilibrium, foster match formation is characterized by a convex set of signals bounded below. That is,*

$$\mathcal{F}(x,\tilde{q}) = [\underbrace{\max\{s^c(x,\tilde{q}),\ s^p(x)\}}_{\textit{threshold signal}},\ \bar{s}]$$

where $s^c(x,\tilde{q})$ and $s^p(x)$ represent the threshold signals at which the child and parent announce foster, respectively. Moreover, ***(i)*** *Children with greater care needs are* ***less likely*** *to form f-matches; that is, $\max\{s^c(x,\tilde{q}),\ s^p(x)\} \downarrow\ x$.* ***(ii)*** *Children are* ***less likely*** *to form a new foster match as the quality of their current match increases; that is, $\max\{s^c(x,q),\ s^p(x)\} \uparrow q$.* ***(iii)*** *For a child currently in an f-match of quality $q \in Q$, the likelihood of forming a* new *foster match is nonincreasing in q; that is, $\max\{s_c(x,q),\ s_p(x)\} \uparrow q$.*

Both children and parents compare the value of forming an f-match to their respective outside options. Because higher signals s imply a more favorable distribution of match quality, the expected continuation value from fostering is increasing in s. Each side therefore adopts a cutoff rule: the child accepts if $s \geq s^c(x,\tilde{q})$ and the parent accepts if $s \geq s^p(x)$. An f-match forms only when both conditions hold, implying that the joint acceptance region is an interval bounded below by $\max\{s^c(x,\tilde{q}), s^p(x)\}$.

Intuition. An f-match forms only when both sides are convinced, from a noisy signal, that the f-match is likely to be good enough to be worth entering—because once formed, the true quality is revealed and both sides are locked into that realization for at least a period before either can act on it. This generates two distinct forces behind the threshold structure. First, because parents expect lower payoffs from fostering children with greater care needs, prospective parents require a stronger signal before agreeing to foster them, regardless of whether the child is currently unmatched or already in an f-match. For unmatched children, this is exactly the higher threshold behind Fact 3: disability lowers the probability of moving from institutional care into an f-match. But the same force also means that children with greater care needs are less likely to switch out of an existing f-match into a new one, since prospective parents remain equally reluctant to accept them irrespective of their current placement status. Second, children already placed in a good f-match become more picky while swapping partners: forming a new f-match means giving up an already known partner for an uncertain one, and thus, a promising high signal makes it worth trading the existing partner. Because forming an f-match requires a mutual consent, the joint threshold is simply the highest of the child's and the parent's individual thresholds, whichever currently binds.

Proposition 2 (**Foster-Match Dissolution**)**.** *In equilibrium, foster match dissolution is characterized by an interval of match qualities bounded above:*

$$\mathcal{D}(x) = [\underline{q},\, \underbrace{\max\{q_d^c(x),\, q_d^p(x)\}}_{\textit{threshold quality}}].$$

*where $q_d^c(x)$ and $q_d^p(x)$ denote the match-quality thresholds at which the child and parent prefer dissolution, respectively. Moreover, **(i)** Children with greater care needs are **more likely** to experience foster match dissolution; that is, $q_d^c(x) \downarrow x$ and $q_d^p(x) \downarrow x$. **(ii)** High-quality foster matches are **less likely** to be dissolved.*

Intuition. Agents choose to destroy an f-match only when the value of returning to the market exceeds both the value of continuing to foster and the value of transitioning to adoption. Both valuations (continue with f-match or transition to a-match) rise with the realized quality q whereas the value of being unmatched does not. Therefore, a threshold strategy is generated: for sufficiently low q, returning to the market dominates both alternatives and the f-match is destroyed; for higher q, either continuing to foster or adopting becomes preferable, and the match survives.

Children with greater care needs raise this cutoff for both parties. A poorer fit with a high-need child is more costly to sustain for the parent, and the same complementarity between match quality and care needs operates on the child's side, so higher-quality realizations matter disproportionately more for children with greater needs. As a result, both the child's and the parent's individual thresholds for destruction rise as care needs increase, and since dissolution occurs whenever either side prefers to leave, the joint dissolution region—the union of the two—expands as care needs rise.

Having characterized foster match formation and dissolution, we now turn to adoption decisions. Adoption applies only to foster matches that survive both earlier stages and therefore generate strictly positive payoffs for both children and parents. As a result, adoption is never compared to dissolution in equilibrium, but rather to continued fostering. This distinction implies that, unlike formation and destruction—which are governed by monotone threshold rules—adoption decisions **need not be monotone in match quality**.

Proposition 3 (**Adoption Decisions**). *In equilibrium, adoption decisions are characterized by a subset of match qualities bounded below. For each $x \in X$, the adoption region can be written as*

$$\mathcal{A}(x) = \bigcup_{k=1}^{K(x)} [\underline{q}_k(x),\, \overline{q}_k(x)] \subseteq [q_a(x), \overline{q}],$$

where $q_a(x)$ denotes the minimum match quality at which adoption becomes feasible. Moreover:

- (***i***) *Children with greater care needs (lower x) are **less likely** to be adopted; equivalently, the minimum feasible adoption quality $q_a(x)$ is nonincreasing in x.*
- (***ii***) *If adoption entails no penalty, the adoption region is a single interval in match quality. Adoption penalties may generate non-convex adoption regions.*

Intuition. The potential non-monotonicity of adoption decisions arises from the interaction between the continuation value of fostering and the adoption penalty. As match quality improves, the continuation value of fostering increases through two channels. First, higher-quality matches generate greater current surplus. Second, they reduce the endogenous probability that the child transitions to another foster family in search of a better match. For sufficiently low match qualities, this transition occurs whenever another foster opportunity arises, so improvements in match quality affect only current surplus. Beyond a threshold, however, higher match quality also lowers the probability of such transitions, causing the continuation value of fostering to increase more rapidly. Once match quality becomes sufficiently high, this endogenous transition probability falls to zero, so further improvements again affect only current surplus. Consequently, the continuation value of fostering becomes steeper over an intermediate range of match qualities and less steep thereafter.

Figure 2 presents the implications of this mechanism. In the absence of an adoption penalty, adoption dominates continued fostering for sufficiently high-quality matches. An adoption penalty shifts the value of adoption downward relative to the continuation value of fostering. Because the continuation value of fostering changes slope as match quality affects endogenous transition probabilities, the two value functions may intersect multiple times, generating a non-convex adoption region in match quality.

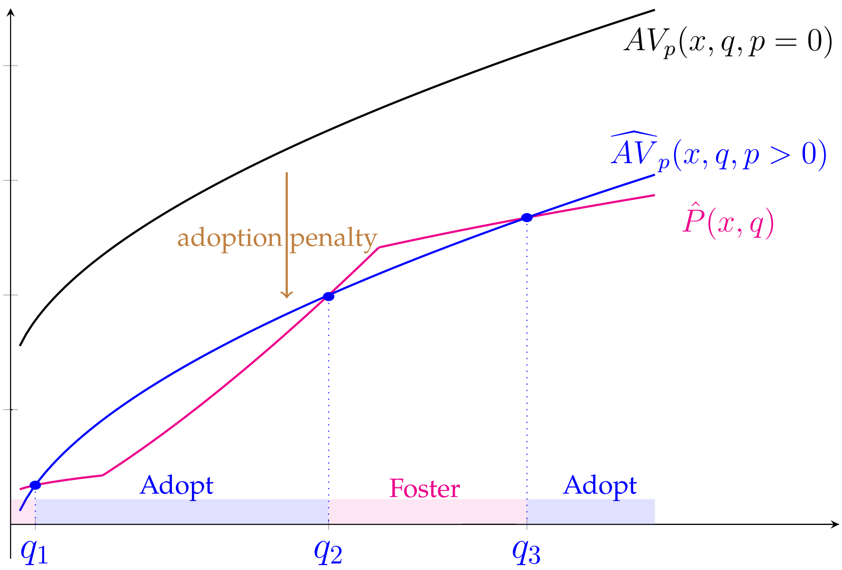

Figure 2: Illustration of adoption decisions with an adoption penalty. The penalty shifts the value of adoption downward relative to the continuation value of fostering, generating multiple intersections and therefore a non-convex adoption region in match quality.

An advantage of the theoretical framework is that it delivers equilibrium predictions over the full distribution of match quality, which is unobserved in the data. In particular, it characterizes which foster matches are more likely to transition to adoption, conditional on surviving earlier stages of the matching process. These quality-dependent predictions cannot be directly tested empirically, but they provide a structural interpretation of the observed adoption and dissolution patterns and highlight that adoption penalties may unintentionally discourage adoption for a subset of otherwise successful foster matches.

5.3 Relation to Stylized Facts

The equilibrium characterization in Section 5.2 explains the four stylized facts through two decision margins: the formation of foster matches (Proposition 1) and the dissolution of foster matches together with adoption decisions (Propositions 2 and 3). The former governs two situations: an unmatched child forming a foster match and a foster-matched child switching to another foster home in search of a better match. Dissolution and adoption decisions determine whether an existing foster match ends, continues, or transitions to adoption. The stylized facts arise from different combinations of these equilibrium decisions.

Table 2 summarizes this mapping. Facts 3 and 4 correspond directly to observed transitions between institutional care and foster homes. By contrast, Facts 1 and 2 combine multiple equilibrium decisions. Adoption depends both on the probability of forming a foster match and on the subsequent decision to transition from fostering to adoption. Similarly, foster-match separations arise both when a child becomes unmatched and when a child leaves one foster home for another. The data observe these transitions but not the underlying decisions that generate them. The theoretical model therefore identifies the mechanisms behind the empirical patterns.

Table 2: Decomposition of the Stylized Facts by Decision Margin

	F-match formation (P1)		Disposition (P2–P3)		
	Formation	Switching	Dissolve	Adopt	Net
Fact 1 – adopted (stock)	$\downarrow$			$\downarrow$	$\downarrow$
Fact 2 – separation (flow)		$\downarrow$	$\uparrow$		$\uparrow$
Fact 3 – becoming f-matched (flow)	$\downarrow$				$\downarrow$
Fact 4 – becoming unmatched (flow)			$\uparrow$		$\uparrow$

Notes: Arrows indicate the effect of disability on each equilibrium margin. The Net column reports the resulting effect on each empirical outcome. Fact 2 combines two margins that operate in opposite directions: disability increases dissolution but reduces switching, with the former dominating

The distinction between observed transitions and equilibrium decisions is important. The empirical analysis documents reduced-form relationships between disability and observed outcomes, whereas the theoretical model decomposes these outcomes into the underlying decisions made by children and parents. This, in turn, allows us to disentangle the equilibrium decisions that jointly determine the observed outcomes and to make predictions for objects that are not directly observed, such as match quality.

First, consider foster-match formation. Proposition 1 implies that children with greater care needs require a stronger signal before a foster match is formed because prospective parents expect lower payoffs from fostering these children. As a result, they are less likely to form a foster match when unmatched, explaining Fact 3. The same force also makes them less likely to leave an existing foster match in search of another foster family, since prospective parents remain less willing to foster them. Thus, Proposition 1 governs both entry into foster care and switching across foster homes.

Next, consider foster-match dissolution. Proposition 2 implies that children with greater care needs are more likely to experience the dissolution of a foster match because both children and parents require a higher realized match quality to continue the relationship. However, the data do not distinguish whether a dissolved foster-match ends with the child becoming unmatched or immediately moving to another foster home. The model separates these possibilities. Children with greater care needs are more likely to experience the first type of transition but less likely to experience the second because they are less likely to form a new foster match. Fact 2 therefore implies that the increase in dissolution dominates the reduction in re-matching.

Finally, Proposition 3 explains why children with greater care needs are less likely to be adopted. Adoption requires both the formation of a foster match and the decision to transition from fostering to adoption. As discussed above, children with greater care needs are already less likely to form foster matches. Conditional on forming a foster match, children with greater care needs are also less likely to transition to adoption. Even absent an adoption penalty, they are less likely to transition to adoption because they generate lower surplus from adoption. The adoption penalty introduces an additional force. As shown in Proposition 2, conditional on the foster match surviving, children with greater care needs are less likely to transition to another foster family in search of a better match. The adoption penalty therefore amplifies this disadvantage, further reducing the likelihood that children with greater care needs transition to adoption.

The model also generates predictions beyond the empirical analysis. Because match quality is not observed in the data, the empirical analysis cannot distinguish how outcomes vary across high- and low-quality foster matches. The equilibrium characterization predicts that high-quality foster matches are less likely to dissolve but, in the presence of an adoption penalty, need not transition monotonically to adoption. In particular, some relatively high-quality foster matches may optimally remain in foster care even though lower-quality matches are adopted. This prediction follows from the non-convex adoption region characterized in Proposition 3. Moreover, this effect is more pronounced for children with greater care needs since, as shown in Proposition 2, conditional on the foster match surviving, they are less likely to transition to another foster family in search of a better match.

6 Concluding Remarks

This paper provides an extensive analysis of match transitions among children relinquished for adoption in the U.S. foster care system. We first present an empirical analysis that yields four new facts. Thereafter, we develop a two-sided search and matching model used to rationalize the empirical facts and carry out predictions regarding match quality.

Using the theoretical model, we show that foster match separation involving children with a disability is mainly driven by the uncertainty of the quality of the match, while foster separation involving children without a disability is driven to improve match quality. Also, we find that high-quality matches are less likely to separate. Surprisingly, we find that foster match separation plays a crucial role in adoption by influencing the incentives of foster parents to adopt. Due to the presence of the financial penalty on adoption, parents face the following trade-off when deciding to adopt: accept the penalty in exchange for eliminating the likelihood that the child breaks the foster match in the future. For adoption, we show that the adoption penalty not only exacerbates the intrinsic disadvantage faced by children with a disability but also creates incentives for high-quality matches to not transit to adoption. Moreover, we show that foster parents in high-quality matches might have fewer incentives to adopt.

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A Appendix: Tables

Table A1: Descriptive Statistics, All Samples

	Sample A $obs = 1,109,958$		Sample B $obs = 668,383$		Sample C $obs = 50,323$	
	Mean	sd	Mean	sd	Mean	sd
Adopted (Exited via Adoption)	0.26	0.44				
Foster-Matched Placement	0.95	0.23	1.00	0.00	0.00	0.00
Becoming Foster Matched					0.27	0.45
Becoming Unmatched			0.02	0.13		
Foster Match Separation			0.18	0.38		
Age (in Years)	7.65	4.37	7.68	4.22	12.59	2.61
Has a Disability	0.36	0.48	0.37	0.48	0.67	0.47
Female	0.47	0.50	0.48	0.50	0.37	0.48
White	0.46	0.50	0.45	0.50	0.48	0.50
Black	0.20	0.40	0.21	0.40	0.22	0.41
Hispanic	0.22	0.41	0.22	0.42	0.19	0.39
Title IV-E Eligible	0.46	0.50	0.49	0.50	0.43	0.49
Time in Foster Care (Months)	39.86	22.14	41.50	21.93	55.55	31.90
Time Since PRT* (Months)	19.53	19.89	20.64	18.75	41.08	33.43
of Which: Ending in Adoption	*12.71*	*11.68*				
Duration of Current Placement (Months)	21.16	17.06	24.78	17.03	16.72	17.47
of Which: Foster-Matched Placement	*21.47*	*17.01*				

Notes: Data are from the Adoption and Foster Care Analysis and Reporting System (AFCARS). Means and standard deviations are calculated at the child-period level. Sample A is the full sample of all children younger than 16 whose parental rights have been terminated and who are either foster matched or unmatched. Samples B and C are subsamples of Sample A. Sample B includes observations where the child is foster matched at the beginning of the period and remains in foster care at the end. Sample C includes observations where the child is unmatched at the beginning of the period and also remains in foster care at the end.
* PRT stands for Parental Rights Terminated.

Table A2: Descriptive Statistics by Disability Status - All Samples

	Sample A $obs = 1,109,958$		Sample B $obs = 668,383$		Sample C $obs = 50,323$	
Disability	Yes	No	Yes	No	Yes	No
Adopted (Exited via Adoption)	0.23	0.28			-	-
	(0.42)	(0.45)				
Foster-Matched Placement	0.90	0.97	1.00	1.00	0.00	0.00
	(0.30)	(0.17)	(0.00)	(0.00)	(0.00)	(0.00)
Becoming Foster Matched					0.26	0.31
					(0.44)	(0.46)
Becoming Unmatched			0.03	0.01		
			(0.17)	(0.09)		
Foster Match Separation			0.18	0.17		
			(0.39)	(0.38)		
Age (in Years)	8.83	6.98	8.72	7.09	12.65	12.48
	(4.37)	(4.23)	(4.18)	(4.12)	(2.50)	(2.80)
Female	0.43	0.50	0.43	0.50	0.35	0.41
	(0.50)	(0.50)	(0.50)	(0.50)	(0.48)	(0.49)
White	0.46	0.47	0.44	0.45	0.49	0.45
	(0.49)	(0.50)	(0.50)	(0.50)	(0.50)	(0.50)
Black	0.21	0.19	0.22	0.20	0.22	0.23
	(0.40)	(0.39)	(0.41)	(0.40)	(0.41)	(0.42)
Hispanic	0.21	0.22	0.22	0.23	0.18	0.21
	(0.41)	(0.42)	(0.41)	(0.42)	(0.38)	(0.41)
Title IV-E Eligible	0.48	0.46	0.51	0.48	0.41	0.47
	(0.50)	(0.50)	(0.50)	(0.50)	(0.49)	(0.50)
Time in Foster Care (Months)	45.50	36.64	46.82	38.45	57.44	51.80
	(25.65)	(19.12)	(25.02)	(19.29)	(32.62)	(30.05)
Time Since PRT* (Months)	24.22	16.84	24.64	18.35	42.85	37.55
	(23.88)	(16.62)	(21.99)	(16.16)	(33.92)	(32.14)
Duration of Current Placement (Months)	22.27	20.52	25.79	24.19	16.78	16.60
	(18.57)	(16.10)	(18.91)	(15.83)	(17.93)	(16.52)

Notes: Data are from the Adoption and Foster Care Analysis and Reporting System (AFCARS). Means and standard deviations are calculated at the child-period level. Sample A is the full sample of all children younger than 16 whose parental rights have been terminated and who are either foster matched or unmatched. Samples B and C are subsamples of Sample A. Sample B includes observations where the child is foster matched at the beginning of the period and remains in foster care at the end. Sample C includes observations where the child is unmatched at the beginning of the period and also remains in foster care at the end.

* PRT stands for Parental Rights Terminated.

B Appendix: Omitted Equations

B.1 Endogenous Distribution of Children

Let $\hat{m}(x,\tilde{q})$ denote the share of children with characteristics $(x,\tilde{q})$ relative to the total mass of children currently in the market. Formally,

$$\hat{m}(x,\tilde{q}) = \frac{m(x,\tilde{q})}{\int_X \left[u^c(x) + \int_Q m(x,q')\,dq'\right] dx}, \tag{B.1}$$

where the unmatched state is captured by defining $m(x,q_0) \equiv u^c(x)$.

The object $\hat{m}(x,\tilde{q})$ represents the density of meeting a child of type $(x,\tilde{q})$ conditional on meeting a child. Therefore, the probability that a parent meets a child in a measurable set $A \subseteq X \times \tilde{Q}$ is

$$\pi^p(\theta) \int_A \hat{m}(x,\tilde{q})\,d\tilde{q}\,dx,$$

where $\pi^p(\theta)$ is the probability that a parent meets a child given market tightness θ.

B.2 Aggregate State of the Market

In a stationary equilibrium, the aggregate state of the market $\phi = (u^p, u^c, m)$ is time-invariant. That is, the distribution of children must remain constant over time, which requires that the flow into each foster-match state (x,q) equals the flow out of that state. In what follows, we define aggregate state of the market, and it proves useful to introduce a couple of notations: Let $d(x,q) = \max\{d^c(x,q), d^p(x,q)\}$ and $a(x,q) = \min\{a^c(x,q), a^p(x,q)\}$ denote the joint destruction and adoption decisions, respectively.

Steady state $\mathbf{m(x,q)}$. Consider an f-match (x,q) at the beginning of a period. With probability δ, a relative arrives and adopts the child, in which case the match is exogenously destroyed. If no relative arrives, the child and the parent simultaneously decide whether to continue the f-match, pursue adoption, or terminate the f-match endogenously. The f-match is dissolved if either agent chooses to end it. If the f-match survives, the child searches for an alternative parent. The child leaves the current match if she finds another parent, observes a signal, and both the child and the prospective parent are willing to form the new f-match. Therefore, the **total mass of f-matches of type** (x,q) **destroyed** in each period is:

$$m(x,q)\left[\delta+(1-\delta)\max\left\{d(x,q),a(x,q),\pi^c(\theta)\int_{\mathcal{F}(x,q)}h(s)\,\mathrm{d}s\right\}\right] \tag{B.2}$$

Now, we specify the creation flow of $m(x,q)$ f-matches. First note that f-matches can be created by unmatched children, by new children who arrive to the market, or by other matches $m(x,q')$ that are formed by children in some f-match $m(x,q')$. At the beginning of a period, an unmatched child is adopted by a relative with probability δ; if she is not adopted, then she searches and can form a new match after observing some signal s, and this match has quality q with probability $g(q \mid s)$. Similarly, some new children arrive at rate ρ and draw a type from the distribution $l(x)$; since these children enter the market unmatched, we assume they make the same decision as an unmatched agent, so they can also search and form a potential f-match (x,q). Lastly, consider an f-match (x,q'): if no relative arrives and neither agent chooses to destroy or adopt, the child searches and may leave the match to form a new one of quality q. Therefore, the **total mass of f-matches of type** (x,q) **created** in each period is:

$$\begin{aligned}u^c(x)(1-\delta)\pi^c(\theta)\int_{\mathcal{F}(x,q_0)}h(s)\,g(q\mid s)\,\mathrm{d}s+\rho l(x)\pi^c(\theta)\int_{\mathcal{F}(x,q_0)}h(s)\,g(q\mid s)\,\mathrm{d}s\\+\int_Q m(x,q')\left[(1-\delta)\big(1-\max\{d(x,q'),a(x,q')\}\big)\pi^c(\theta)\int_{\mathcal{F}(x,q')}h(s)\,g(q\mid s)\,\mathrm{d}s\right]\mathrm{d}q'.\end{aligned} \tag{B.3}$$

In a stationary equilibrium, Equations (B.2) and (B.3) must balance for all (x,q).

Steady state $\mathbf{u^c(x)}$. Consider a child x who is unmatched at the beginning of a period. Every period, new children arrive unmatched, and they remain unmatched if they fail to form a new f-match in the search stage. Similarly, a child who starts the period in an f-match (x,q) contributes to $u^c(x)$ if she is not adopted by a relative, the f-match is destroyed rather than continued or transitioned to adoption, and she subsequently fails to form a new f-match in the search stage. Thus, the **total mass of new unmatched**

children of type x in each period is:

$$\rho l(x)\Big[1-\pi^c(\theta)\int_{\mathcal{F}(x,q_0)} h(s)\,\mathrm{d}s\Big]+\int_Q m(x,q)(1-\delta)d(x,q)\Big[1-\pi^c(\theta)\int_{\mathcal{F}(x,q_0)} h(s)\,\mathrm{d}s\Big]\mathrm{d}q$$

Next, consider a child x who is unmatched at the beginning of a period. With probability δ, a relative arrives and adopts the child, in which case she leaves the pool of unmatched children. If no relative arrives, the child searches for a parent in the search stage and leaves $u^c(x)$ if she meets a parent, observes a signal, and both she and the prospective parent are willing to form a new f-match. Thus, the **total mass of unmatched children of type** x **destroyed** in each period is:

$$u^c(x)\left[\delta+(1-\delta)\pi^c(\theta)\int_{\mathcal{F}(x,q_0)} h(s)\,\mathrm{d}s\right]$$

In a stationary equilibrium, Equations (B.2) and (B.2) must balance for all x.

Steady state $\mathbf{u^p}$. The equilibrium market tightness θ^* is pinned down by the free-entry condition, which binds at zero, $P^u = 0$. Since $u^c(x)$and $m(x,q)$ are time-invariant by the flow-balance conditions established above, and since, by the definition of market tightness,

$$u^p=\theta^*\left[\int_X\left\{u^c(x)+\int_Q m(x,q)\,dq\right\}dx\right],$$

u^p is uniquely recovered from θ^* and the (already stationary) distribution of children. Hence u^p is time-invariant as well.

C Appendix: Definition of Equilibrium

Definition C.1. *A **foster-care equilibrium** is a collection*

$$\Big(\underbrace{\hat{\mathcal{C}}(x,\tilde{q}),\,\mathcal{C}(x,q_0),\,\mathcal{C}(x,q),\,\hat{\mathcal{P}}(x,q),\,\mathcal{P}^u,\,\mathcal{P}(x,q)}_{\textit{value functions}};\,\underbrace{\mathcal{F},\,d^c,\,d^p,\,a^c,\,a^p,\,in}_{\textit{strategies}};\,\underbrace{u^p,u^c,m}_{\textit{aggregate state}}\Big)$$

such that the following properties are satisfied.

(1) ***Value functions.*** *Given the strategies and the aggregate state, the child value functions* $\hat{\mathcal{C}}(x,\tilde{q})$, $\mathcal{C}(x,q_0)$, $\mathcal{C}(x,q)$ *are determined by Equations 2–4; analogously, the parent value*

functions $\hat{\mathcal{P}}(x, q)$, $\mathcal{P}^u$, $\mathcal{P}(x, q)$ *are determined by Equations 5–7.*

(2) ***Strategies.*** *Given the value functions and the aggregate state, individual strategies are optimal. In particular:*

- **(*i*)** *Children's foster-matching, adoption, and destruction decisions are determined by Lemma 1.*
- **(*ii*)** *Parents' entry, foster-matching, adoption, and destruction decisions are determined by Lemma 2.*

(3) ***Aggregate state of the market.*** *Given the value functions and the strategies, the aggregate state is stationary, as characterized in Appendix B.2. In particular:*

- (a) ***Free entry of parents.*** *By Lemma 2, individual entry is optimal whenever* $P^u \geq 0$. *Consistency of this condition with a finite, strictly positive mass of entrants* $u^p > 0$ *requires* $P^u = 0$ *in equilibrium: if* $P^u > 0$ *held instead, every unmatched parent outside the market would strictly prefer to enter, so* u^p *could not remain at an interior stationary level. Hence* $P^u = 0$*, and the equilibrium market tightness* θ^* *satisfies:*

$$\pi^p(\theta^*) = \frac{k}{\beta \int_X \int_{\tilde{Q}} \hat{m}(x, \tilde{q}) \int_{s \in F(x,\tilde{q})} E_s[P(x, q)]\, h(s)\, ds\, d\tilde{q}\, dx}.$$

- (b) ***Flow balance of children.*** *For each* $x \in X$ *and each* $q \in Q$*, the distribution of foster matches* $m(x, q)$ *satisfies the steady-state flow-balance condition equating the mass of* (x, q)*-matches destroyed and created each period, as defined in Appendix B.2. Likewise, for each* $x \in X$*, the mass of unmatched children* $u^c(x)$ *satisfies the analogous flow-balance condition equating mass entering and leaving unmatched status, as defined in the same appendix.*

- (c) ***Level of unmatched parents.*** *Given the equilibrium tightness* θ^* *from part* (a) *and the stationary distribution* (u^c, m) *from part* (b)*, the mass of unmatched parents satisfies*

$$u^p = \theta^* \left[\int_X \left\{ u^c(x) + \int_Q m(x, q)\, dq \right\} dx \right],$$

consistent with the definition of market tightness and the steady state of u^p *derived in Appendix B.2. Since* θ^*, $u^c(x)$*, and* $m(x, q)$ *are time-invariant,* u^p *is time-invariant as well.*

D Appendix: Proofs

D.1 Decisions to Foster, Destroy and Adopt

Proof of Lemma 1. All statements follow from one-step optimality at the relevant decision nodes.

(i) Foster. After observing signal s, a child $(x, \tilde{q})$ compares rejecting (value $\mathcal{C}(x, \tilde{q})$) to accepting a new foster match (value $\mathbb{E}_s[\mathcal{C}(x, q')]$). Hence

$$f^c(x, \tilde{q}, s) = 1 \iff \mathbb{E}_s[\mathcal{C}(x, q')] \geq \mathcal{C}(x, \tilde{q}).$$

(ii) Adoption. Conditional on not destroying, adoption yields $\frac{b^c(x,q,a)}{1-\beta}$ and the alternatives are to continue in the market as unmatched or f-matched, with values $\hat{\mathcal{C}}(x, q_0)$ and $\hat{\mathcal{C}}(x, q)$, respectively. Thus:

$$a^c(x, q) = 1 \iff \frac{b^c(x, q, a)}{1 - \beta} \geq \max\{\hat{\mathcal{C}}(x, q_0), \hat{\mathcal{C}}(x, q)\}.$$

(iii) Destruction. Destruction yields $\hat{\mathcal{C}}(x, q_0)$; the other feasible continuation values are adoption and remaining f-matched. Therefore

$$d^c(x, q) = 1 \iff \hat{\mathcal{C}}(x, q_0) > \max\left\{\frac{b^c(x, q, a)}{1 - \beta}, \hat{\mathcal{C}}(x, q)\right\}.$$

□

Proof of Lemma 2. Each statement follows from optimal comparisons of continuation values at the relevant decision node, as in the earlier section.

(i) Entry. A parent outside the market enters iff the value of entering, $\mathcal{P}^u$, weakly exceeds the outside option 0, i.e. iff $\mathcal{P}^u \geq 0$.

(ii) Foster. Upon meeting a child of type x and observing signal s, accepting yields expected continuation value $\mathbb{E}_s[\mathcal{P}(x, q)]$, while rejecting yields $\mathcal{P}^u$. Hence $f^p(x, s) = 1$ iff $\mathbb{E}_s[\mathcal{P}(x, q)] \geq \mathcal{P}^u$.

(iii) Adoption and destruction. In an f-match (x, q), adoption yields $\frac{b^p(x,q,a)}{1-\beta}$, destroying yields $\mathcal{P}^u$, and remaining yields $\hat{\mathcal{P}}(x, q)$. Therefore $a^p(x, q) = 1$ if and only if $\frac{b^p(x,q,a)}{1-\beta} \geq \max\{\mathcal{P}^u, \hat{\mathcal{P}}(x, q)\}$, and $d^p(x, q) = 1$ if and only if $\mathcal{P}^u > \max\{\frac{b^p(x,q,a)}{1-\beta}, \hat{\mathcal{P}}(x, q)\}$.

$\square$

D.2 Monotonicity in match quality q

Lemma D.1 (**Monotonicity in Match Quality,** q)**.** *Maintain Assumptions 1 and 2. Then, in a stationary equilibrium, the value functions $\mathcal{C}(x,q)$, $\hat{\mathcal{C}}(x,q)$, $\mathcal{P}(x,q)$, and $\hat{\mathcal{P}}(x,q)$ are nondecreasing in q on Q.*

Proof of Lemma D.1. We prove the result for children. The proof for parents is analogous. Fix x.

Step 1: Rewrite the f-matched child value $\mathcal{C}(x,q)$.

Using Lemma 1, the continuation value of a child in an f-match (x,q) after the destruction/adoption stage equals

$$\max\left\{\hat{\mathcal{C}}(x,q_0), \frac{b^c(x,q,a)}{1-\beta}, \hat{\mathcal{C}}(x,q)\right\}.[19]$$

Hence the matched value function can be written as

$$\mathcal{C}(x,q) = b^c(x,q,f) + \beta\delta\frac{b^c(x,\overline{q},a)}{1-\beta} + \beta(1-\delta)\cdot\kappa, \tag{D.1}$$

where $\kappa \in \left\{\hat{\mathcal{C}}(x,q_0), \frac{b^c(x,q,a)}{1-\beta}, \hat{\mathcal{C}}(x,q)\right\}$.

Step 2: $\mathcal{C}(x,q)$ is nondecreasing in q if $\hat{\mathcal{C}}(x,q)$ is.

Assume $\hat{\mathcal{C}}(x,q)$ is nondecreasing in q. By assumption 1, both $b^c(x,q,f)$ and $b^c(x,q,a)$ are nondecreasing in q, so $\frac{b^c(x,q,a)}{1-\beta}$ is nondecreasing. The term $\hat{\mathcal{C}}(x,q_0)$ is constant in q. The maximum of nondecreasing functions is nondecreasing. Since all coefficients in (D.1) are nonnegative, $\mathcal{C}(x,q)$ is nondecreasing in q.

Step 3: $\hat{\mathcal{C}}(x,q)$ is nondecreasing in q if $\mathcal{C}(x,q)$ is. The search-stage value of a child (x,q) can be written as follows:

$$\hat{\mathcal{C}}(x,q) = \frac{1}{1-\beta(1-\delta)}\Big(b^c(x,q,f) + \beta\delta\frac{b^c(x,\overline{q},a)}{1-\beta} + \underbrace{\pi^c(\theta)\int_{s\in\mathcal{F}(x,q)}\big[\mathbb{E}C(x,s) - \mathcal{C}(x,q)\big]h(s)\mathrm{d}s}_{R(x,q)}\Big), \tag{D.2}$$

[19] When the parent destroys the foster match, the child transitions to the unmatched state and receives $\hat{\mathcal{C}}(x,q_0)$. Since this continuation value does not depend on the current match quality q, it plays no role in the comparative statics with respect to q considered here.

where $\mathbb{E}C(x,s) \equiv \mathbb{E}_s\big[\mathcal{C}(x,q')\big] = \int_Q \mathcal{C}(x,q')g(q'|s)\mathrm{d}q'$.

By assumption (ii) and the fact that $\mathcal{C}(x,q')$ is nondecreasing in q', the mapping $\mathbb{E}_s[\mathcal{C}(x,q')]$ is nondecreasing in s. Moreover, letting $F^c(x,q) \equiv \{s : \mathbb{E}_s[\mathcal{C}(x,q')] \geq \mathcal{C}(x,q)\}$, the monotonicity of $\mathcal{C}(x,q)$ in q implies $F^c(x,q_2) \subseteq F^c(x,q_1)$ for $q_2 > q_1$, and therefore $\mathcal{F}(x,q_2) \subseteq \mathcal{F}(x,q_1)$ since $\mathcal{F}(x,q) = F^c(x,q) \cap F^p(x)$ and $F^p(x)$ does not depend on q.

Let $q_2 > q_1$. The difference in search-stage values follows:

$$\begin{aligned}\hat{\mathcal{C}}(x,q_2) - \hat{\mathcal{C}}(x,q_1) &= \underbrace{\Big(\mathcal{C}(x,q_2) - \mathcal{C}(x,q_1)\Big)}_{\geq 0}\Big[1 - \pi^c(\theta)\int_{s\in\mathcal{M}(x,q_1)} h(s)\mathrm{d}s\Big] \\ &+ \int_{s\in\mathcal{M}(x,q_1)\setminus\mathcal{M}(x,q_2)} \underbrace{\Big(\mathcal{C}(x,q_2) - \mathbb{E}C(x,s)\Big)}_{\geq 0} h(s)\mathrm{d}s\end{aligned}$$

The first inequality follows from the fact that $b^c(x,\cdot,f)$ is increasing in q, whereas the second inequality is by definition: $s \notin \mathcal{M}(x,q_2)$ implies that $\mathcal{C}(x,q_2) \geq \mathbb{E}C(x,s)$. This finishes the proof of the monotonicity of $\hat{\mathcal{C}}(x,q)$ in q.

Step 4: Close the loop. The Bellman operator maps nondecreasing functions into nondecreasing functions and is a contraction under $\beta < 1$, so its unique fixed point is nondecreasing in q. □

Lemma D.2 (Option value decreases in q). *Maintain Lemma D.1. For each $x \in X$, the option term*

$$R(x,q) = \pi^c(\theta)\int_{s\in\mathcal{F}(x,q)} \big[\mathbb{E}C(x,s) - \mathcal{C}(x,q)\big]h(s)ds$$

is nonincreasing in q.

Proof. Fix $q_2 > q_1$. By Lemma D.1, $\mathcal{C}(x,q_2) \geq \mathcal{C}(x,q_1)$.

Moreover the acceptance region satisfies $\mathcal{F}(x,q_2) \subseteq \mathcal{F}(x,q_1)$ because the child's continuation value increases in q.

Hence both the integration domain shrinks and the integrand $\mathbb{E}C(x,s) - \mathcal{C}(x,q)$ weakly decreases, implying $R(x,q_2) \leq R(x,q_1)$. □

Implications for foster match formation. The previous lemmas imply the structure of foster match formation and its comparative statics with respect to current match quality.

Corollary D.1 (**Foster formation and comparative statics in** q)**.** *In equilibrium, a foster match forms if and only if both agents accept. Hence the set of signals leading to foster formation is an upper interval*

$$\mathcal{F}(x, \tilde{q}) = \left[\max\{s^c(x, \tilde{q}),\ s^p(x)\}, \bar{s}\right].$$

Moreover, the child's acceptance threshold is nondecreasing in the current match quality q, so the foster formation set shrinks in q: for any $q_2 > q_1$,

$$\mathcal{F}(x, q_2) \subseteq \mathcal{F}(x, q_1).$$

Corollary D.1 establishes the interval structure of foster match formation and the comparative statics with respect to match quality stated in Proposition 1 in the main text.

D.3 Monotonicity in child type x

In this subsection we provide primitive sufficient conditions under which the remaining comparative static in Proposition 1 holds: the joint foster-acceptance set $\mathcal{F}(x, \tilde{q})$ expands in x (equivalently, the joint cutoff $\max\{s^c(x, \tilde{q}), s^p(x)\}$ is nonincreasing in x).

Assumption D.1 (**Monotone likelihood ratio property** (**MLRP**))**.** *The family of conditional densities $\{g(\cdot \mid s)\}_{s \in S}$ satisfies MLRP on Q: for any $s' > s$, the likelihood ratio $g(q \mid s')/g(q \mid s)$ is nondecreasing in q.*

Assumption D.2 (**Increasing differences in** (x, q))**.** *For each $z \in \{f, a\}$, the flow payoffs $b^p(x, q, z)$ and $b^c(x, q, z)$ have* increasing differences *in (x, q).*

Lemma D.3 (**Individual acceptance sets expand in** x)**.** *Maintain Lemmas 1–D.1 and Assumptions D.1–D.2. Fix $\tilde{q} \in \tilde{Q}$. Then*

$$F^p(x) = [s^p(x), \bar{s}] \quad \textit{and} \quad F^c(x, \tilde{q}) = [s^c(x, \tilde{q}), \bar{s}]$$

expand in x (set inclusion order). Equivalently, $s^p(x)$ and $s^c(x, \tilde{q})$ are nonincreasing in x.

Proof. We show the claim for parents; the child argument is identical.

Define the parent's net gain from accepting after signal s as

$$\Delta^p(x,s) \equiv \mathbb{E}_s[\mathcal{P}(x,q')] - \mathcal{P}^u.$$

Under Assumption D.2, the flow payoff $b^p(x,q,z)$ has increasing differences in (x,q). In dynamic programming problems with monotone transitions and discounting, increasing differences in the flow payoff are preserved by the Bellman operator. Hence the value function $\mathcal{P}(x,q)$ inherits increasing differences in (x,q) (see Topkis (1998); Stokey and Lucas (1989); Milgrom and Shannon (1994)). Under Assumption D.1, the mapping $G(\cdot \mid s)$ preserves increasing differences when taking expectations of increasing-differences functions. Hence $\Delta^p(x,s)$ has increasing differences in (x,s). By monotone comparative statics (Topkis, 1998), the upper contour set $F^p(x) = \{s : \Delta^p(x,s) \geq 0\}$ expands in x, which is equivalent to $s^p(x)$ being nonincreasing in x since $F^p(x)$ is an upper interval in s. The child case follows by replacing $\mathcal{P}$ with $\mathcal{C}$ and $\mathcal{P}^u$ with $\mathcal{C}(x,\tilde{q})$. □

Corollary D.2 (**Joint acceptance expands in** x)**.** *Maintain Lemmas 1–D.1 and Assumptions D.1–D.2. Then for each $\tilde{q} \in \tilde{Q}$ the joint foster-acceptance set*

$$\mathcal{F}(x,\tilde{q}) = F^c(x,\tilde{q}) \cap F^p(x) = \left[\max\{s^c(x,\tilde{q}), s^p(x)\}, \bar{s}\right]$$

expands in x (set inclusion order). Equivalently, the joint cutoff $\max\{s^c(x,\tilde{q}), s^p(x)\}$ is nonincreasing in x.

Proof. Immediate from Lemma D.3: the intersection of sets that expand in x also expands in x; for upper intervals this is equivalent to the cutoff being nonincreasing. □

Remark D.1 (**Benchmark: child payoff independent of** x)**.** *If $b^c(x,q,z) \equiv b^c(q,z)$ for all (x,q,z), then b^c satisfies (in a weak sense) increasing differences in (x,q) automatically, since it does not vary with x. In that benchmark, $F^c(x,\tilde{q})$ is independent of x and the expansion of $\mathcal{F}(x,\tilde{q})$ in x is driven entirely by $F^p(x)$.*

Implications for foster match dissolution. The same arguments used for foster formation imply a threshold structure for dissolution decisions in match quality and deliver the comparative statics used in Proposition 2.

Corollary D.3 (Dissolution set and comparative statics). *For each $x \in X$, define the individual dissolution regions*

$$D^c(x) \equiv \{q \in Q: \ d^c(x,q) = 1\}, \qquad D^p(x) \equiv \{q \in Q: \ d^p(x,q) = 1\},$$

where $d^c(x,q)$ and $d^p(x,q)$ are given by Lemma 1(iii) and Lemma 2(iii). Then:

(*i*) ***Threshold structure in q.*** *Maintain Lemma D.1. There exist cutoffs $q_d^c(x) \in Q$ and $q_d^p(x) \in Q$ such that*

$$D^c(x) = [\underline{q},\ q_d^c(x)] \quad \textit{and} \quad D^p(x) = [\underline{q},\ q_d^p(x)].$$

(*ii*) ***Joint dissolution set.*** *A match dissolves whenever either side prefers dissolution, hence*

$$\mathcal{D}(x) = D^c(x) \cup D^p(x) = [\underline{q},\ \max\{q_d^c(x), q_d^p(x)\}].$$

In particular, higher-quality matches are less likely to be dissolved.

(*iii*) ***Comparative statics in child type x.*** *Under the primitive conditions for monotone comparative statics in child type (Assumptions D.1–D.2), both dissolution thresholds are nonincreasing in x:*

$$q_d^c(x) \downarrow x \qquad \textit{and} \qquad q_d^p(x) \downarrow x.$$

Equivalently, the joint dissolution set $\mathcal{D}(x)$ expands as care needs rise (that is, as x decreases).

Proof Sketch. First part follows because Lemma D.1 implies that the value of continuing in an f-match is nondecreasing in q, while the relevant outside options from dissolution (Lemma 1(iii) and Lemma 2(iii)) do not depend on q, yielding single crossing and hence a cutoff in q. Second part is immediate from the union of two lower sets in q. Last part follows from the same Topkis/MLRP logic used for foster acceptance: under Assumptions D.1–D.2, the net gain from remaining matched has increasing differences in (x,q), so the quality cutoff required to sustain the match is weakly lower for higher x.

D.4 Adoption: individual and joint regions

For an f-match (x, q) that is not destroyed, define the individual adoption regions

$$A^c(x) \equiv \{q \in Q : \; a^c(x,q) = 1\}, \qquad A^p(x) \equiv \{q \in Q : \; a^p(x,q) = 1\},$$

where $a^c(x,q)$ and $a^p(x,q)$ are given by Lemma 1(ii) and Lemma 2(iii), respectively. The joint adoption region is

$$\mathcal{A}(x) = A^c(x) \cap A^p(x).$$

Assumption D.3 (**Child supermodularity in** (q,z))**.** *For each $x \in X$, the child's flow payoff $b^c(x,q,z)$ has increasing differences in (q,z) for $z \in \{f,a\}$. Equivalently,*

$$b^c(x,q,a) - b^c(x,q,f)$$

is nondecreasing in q.

Remark D.2 (Auxiliary objects for child adoption decisions)**.** *For later use, define the child's option value from continued search as*

$$R(x,\tilde{q}) \equiv \pi^c(\theta) \int_{s \in \mathcal{F}(x,\tilde{q})} \big[\mathbb{E}C(x,s) - \mathcal{C}(x,\tilde{q})\big] h(s)\, ds,$$

which measures the expected gain from remaining in foster care while retaining the possibility of forming a better foster match in the future.

Also define the child's adoption gain

$$\Delta^c_a(x,q) \equiv \frac{b^c(x,q,a)}{1-\beta} - \hat{\mathcal{C}}(x,q).$$

Using the representation of $\hat{\mathcal{C}}(x,q)$, this can be written as

$$\Delta^c_a(x,q) = b^c(x,q,a) - w_f\,[b^c(x,q,f) + R(x,q)] - w_a\, b^c(x,\overline{q},a),$$

for constants $w_f, w_a \in (0,1)$ with $w_f + w_a = 1$.

Lemma D.4 (**Child adoption cutoff**)**.** *Maintain Lemma D.1 and Assumption D.3. Then* *for each $x \in X$ there exists $q_a^c(x) \in [\underline{q}, \overline{q}]$ such that*

$$A^c(x) = [q_a^c(x), \overline{q}].$$

Proof. By Lemma 1(ii), adoption occurs if and only if

$$\Delta_a^c(x, q) \geq 0.$$

Using the representation of $\Delta_a^c(x, q)$ in the auxiliary definition block, Assumption D.3 implies that $b^c(x, q, a) - w_f b^c(x, q, f)$ is nondecreasing in q. By Lemma D.2, the option value $R(x, q)$ is nonincreasing in q. Therefore $\Delta_a^c(x, q)$ is nondecreasing in q.

Hence the superlevel set

$$A^c(x) = \{q : \Delta_a^c(x, q) \geq 0\}$$

is an upper interval, implying the existence of $q_a^c(x) \in [\underline{q}, \overline{q}]$ such that

$$A^c(x) = [q_a^c(x), \overline{q}].$$

□

Assumption D.4 (**Adoption penalty**)**.** *For every (x, q),*

$$b^p(x, q, a) \leq b^p(x, q, f).$$

Equality $b^p(x, q, a) = b^p(x, q, f)$ corresponds to the benchmark with no adoption penalty, *while a strict inequality $b^p(x, q, a) < b^p(x, q, f)$ represents an* adoption penalty.

Remark D.3 (**Constant penalty benchmark**)**.** *A common specification is a constant adoption penalty $p(x) \geq 0$ that depends only on observable characteristics of the child,*

$$b^p(x, q, a) = b^p(x, q, f) - p(x).$$

In this case adoption reduces the parent's flow payoff by $p(x)$ relative to continued fostering, independently of match quality.

Remark D.4 (Auxiliary objects for parent continuation values)**.** *For later use in the analysis of adoption decisions, define*

$$\Pi_c(x,\tilde{q}) \equiv \pi^c(\theta)\int_{\mathcal{F}(x,\tilde{q})} h(s)\,ds,$$

which represents the probability that the child leaves the current status $\tilde{q} \in Q \cup \{q_0\}$ through continued search in the next period.

Define also

$$\zeta^p(x,q) \equiv \frac{1-\Pi_c(x,q)}{1-\beta(1-\delta)(1-\Pi_c(x,q))},$$

so that the parent's continuation value from remaining in a foster match can be written as

$$\hat{\mathcal{P}}(x,q) = \zeta^p(x,q)\,b^p(x,q,f).$$

The multiplier $\zeta^p(x,q)$ therefore captures the discounted expected duration of the foster relationship, taking into account both exogenous separation and the possibility that the child exits through search.

Finally, define the multiplier gap

$$\Gamma(x,q) \equiv \frac{1}{1-\beta} - \zeta^p(x,q),$$

which measures the difference between the lifetime multiplier from permanent adoption and the expected continuation multiplier from remaining in the foster match.

Under the constant-penalty benchmark

$$b^p(x,q,a) = b^p(x,q,f) - p(x),$$

the parent's adoption gain can then be written as

$$\Delta^p_a(x,q) \equiv \frac{b^p(x,q,a)}{1-\beta} - \hat{\mathcal{P}}(x,q) = \Gamma(x,q)\,b^p(x,q,f) - \frac{p(x)}{1-\beta}.$$

Lemma D.5 (**Parent adoption under no penalty**)**.** *Consider the benchmark case of no adoption penalty. Then, for each(x,q), if the parent does not destroy the foster match, the parent* *adopts:*

$$d^p(x,q) = 0 \implies a^p(x,q) = 1.$$

Equivalently, on the set of match qualities for which the parent prefers not to dissolve, the parent never chooses continued fostering over adoption. If in addition the parent's no-destruction region is characterized by the cutoff $q_d^p(x)$ (as in Corollary D.3), then

$$A^p(x) = \{q : d^p(x,q) = 0\} = (q_d^p(x), \overline{q}].$$

Proof. Fix (x,q). By Lemma 2(iii), the parent compares three continuation values: adoption $\frac{b^p(x,q,a)}{1-\beta}$, continued fostering $\hat{\mathcal{P}}(x,q)$, and destruction $\mathcal{P}^u$.

Since $\hat{\mathcal{P}}(x,q) = \zeta^p(x,q)b^p(x,q,f)$ and $\zeta^p(x,q) \leq \frac{1}{1-\beta}$, it follows under no adoption penalty that

$$\frac{b^p(x,q,a)}{1-\beta} = \frac{b^p(x,q,f)}{1-\beta} \geq \hat{\mathcal{P}}(x,q).$$

Intuitively, continued fostering preserves the possibility that the child exits in future periods through continued search or adoption by a relative, whereas adoption removes this source of separation. Thus the parent adopts whenever the match is not destroyed.

Finally, Corollary D.3 implies the parent's destruction region is a lower interval $D^p(x) = [\underline{q}, q_d^p(x)]$. Hence the complement (the no-destruction region) coincides with the parent's adoption region, giving $A^p(x) = (q_d^p(x), \overline{q}]$. □

Corollary D.4 (Joint adoption region without adoption penalty). *Maintain Lemma D.4 and Lemma D.5. Then for each $x \in X$ the joint adoption region is an upper interval:*

$$\mathcal{A}(x) = [q_a(x), \overline{q}], \qquad q_a(x) = \max\{q_a^c(x), q_d^p(x)\}.$$

Proof. By Lemma D.4, $A^c(x) = [q_a^c(x), \overline{q}]$. By Lemma D.5, conditional on not destroying the parent adopts. Since the parent destroys the match for $q \leq q_d^p(x)$ (Corollary D.3), the parent's adoption region is $(q_d^p(x), \overline{q}]$. Hence the joint adoption region is the intersection of two upper intervals,

$$[q_a^c(x), \overline{q}] \cap (q_d^p(x), \overline{q}] = [\max\{q_a^c(x), q_d^p(x)\}, \overline{q}].$$

□

Lemma D.6 (**Parent adoption with an adoption penalty: possible non-convexity**). *Maintain Lemma D.1, Corollary D.1, and the constant-penalty benchmark stated above. Consider matches that are not destroyed.*

Then the parent's adoption region is a closed subset of Q, and hence can be written as a union of closed intervals. In general, $A^p(x)$ need not be a single interval.

Proof. By Lemma 2(iii), conditional on not destroying, the parent adopts if and only if $\Delta_a^p(x,q) \geq 0$.

By Corollary D.1, $\mathcal{F}(x,q)$ shrinks in q. Hence $\Pi_c(x,q)$ is nonincreasing in q, so $\zeta^p(x,q)$ is nondecreasing in q, and therefore $\Gamma(x,q)$ is nonincreasing in q.

Since $b^p(x,q,f)$ is nondecreasing in q by maintained payoff monotonicity, the adoption gain

$$\Delta_a^p(x,q) = \Gamma(x,q)\, b^p(x,q,f) - \frac{p(x)}{1-\beta}$$

is the sum of a product of a nonincreasing term and a nondecreasing term, minus a constant. Such an expression need not be monotone in q. Therefore the superlevel set

$$A^p(x) = \{q : \Delta_a^p(x,q) \geq 0\}$$

need not be an interval.

Since $\Delta_a^p(x,q)$ is a real-valued function of q, the set $A^p(x)$ is a closed subset of Q, and therefore can be expressed as a union of closed intervals. □

Corollary D.5 (**Joint adoption region with an adoption penalty**). *Maintain Lemma D.4 and Lemma D.6. Then for each $x \in X$ the joint adoption region*

$$\mathcal{A}(x) \subseteq [q_a(x), \overline{q}], \qquad q_a(x) = \max\{q_a^c(x), q_a^p(x)\},$$

where $q_a^p(x) \equiv \inf A^p(x)$ whenever $A^p(x) \neq \emptyset$.

Lemma D.7 (**Parent adoption threshold across child types**). *Maintain Corollary D.2 and the constant-penalty benchmark stated above. Suppose moreover that $p(x)$ is nonincreasing in x. Then, for each fixed $q \in Q$, the parent's adoption gain $\Delta_a^p(x,q)$ is nondecreasing in x. Consequently, there exists a threshold $x^p(q) \in [\underline{x}, \overline{x}]$ such that, conditional on the match not being destroyed,*

$$a^p(x,q) = 1 \iff x \geq x^p(q).$$

Proof. Fix $q \in Q$. By Corollary D.2, $\mathcal{F}(x, q)$ expands in x. Hence $\Pi_c(x, q)$ is nondecreasing in x, so $\zeta^p(x, q)$ is nonincreasing in x, and therefore $\Gamma(x, q)$ is nondecreasing in x.

By Assumption 1, $b^p(x, q, f)$ is nondecreasing and nonnegative in x. Since $p(x)$ is nonincreasing in x, the adoption gain

$$\Delta_a^p(x, q) = \Gamma(x, q)\, b^p(x, q, f) - \frac{p(x)}{1 - \beta}$$

is nondecreasing in x.

By Lemma 2(iii), conditional on not destroying, the parent adopts if and only if $\Delta_a^p(x, q) \geq 0$. Hence the set of types for which adoption occurs is an upper interval in x, implying the existence of a threshold $x^p(q)$. □

Corollary D.6 (**Children with greater care needs are less likely to be adopted**)**.** *Maintain Lemma D.4, Corollary D.5, and Lemma D.7. Then children with greater care needs (lower x) are weakly less likely to be adopted.*

Proof. By Lemma D.4, the child's adoption region is an upper interval in q. By Lemma D.7, for each fixed q the parent's adoption region is an upper interval in x. Hence, for any fixed match quality q, the set of child types for which adoption occurs expands in x. Equivalently, children with greater care needs (lower x) are weakly less likely to be adopted. □

Remark D.5. *A sufficient primitive condition for the penalty monotonicity assumption is that $b^p(x, q, z)$ has increasing differences in (x, z), in which case the implied adoption penalty*

$$p(x, q) \equiv b^p(x, q, f) - b^p(x, q, a)$$

is nonincreasing in x for each fixed $q \in Q$.